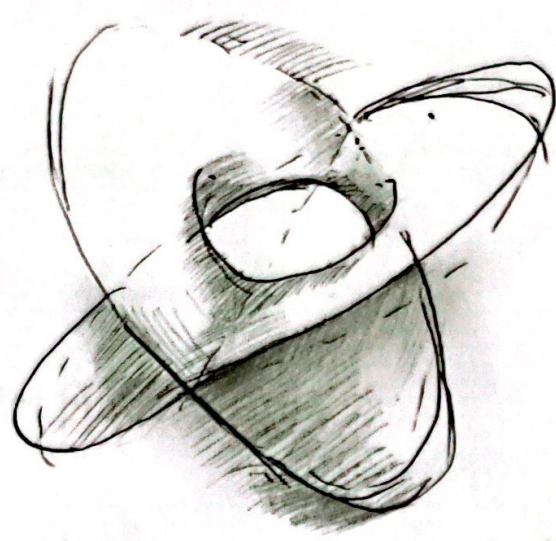


k^3 surface.



• The Milnor number of the singularity.

• A "deformation" $f^+(s) \rightarrow f^+(0)$ ($s \neq 0$) and the Poincaré duality.

• The Picard / transcendental lattice of the family

$\pi^* \mathcal{O}(1) \otimes \mathcal{O}(k)$

$\pi^* \mathcal{O}(1) \otimes \mathcal{O}(k)$

k^3 surface. 代数几何

Remark.

◦ The Milnor number of the singularity

◦ A "deformation" $f^{-1}(s) \rightarrow f^{-1}(0)$ ($s \neq 0$).

and the Poincaré duality.

◦ The Hodge-Poincaré/transcendental lattice of the family.

\mathbb{Q} : let X be a simple k^3 singularity

Thm 1.1.

There exists a quasi-homogeneous

polynomial f with weight system s, τ .

$$X = (f=0)$$

$$f \in \mathbb{C}[z_1, \dots, z_4]$$

$$f(\zeta_1^{q_1} z_1, \dots, \zeta_4^{q_4} z_4) = \zeta^p f(z_1, \dots, z_4)$$

$$d = q_1 + q_2 + q_3 + q_4$$

deg

Δ is reflexive \iff the toric variety \mathbb{P}_Δ is smooth (3 fold)

Belcarin

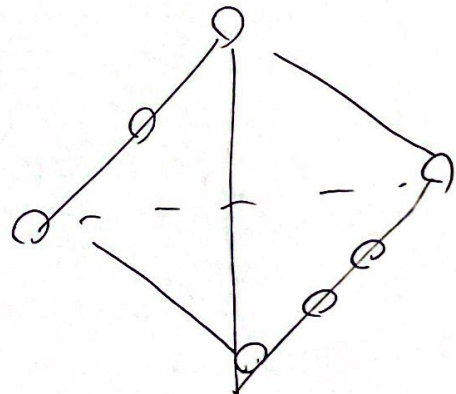
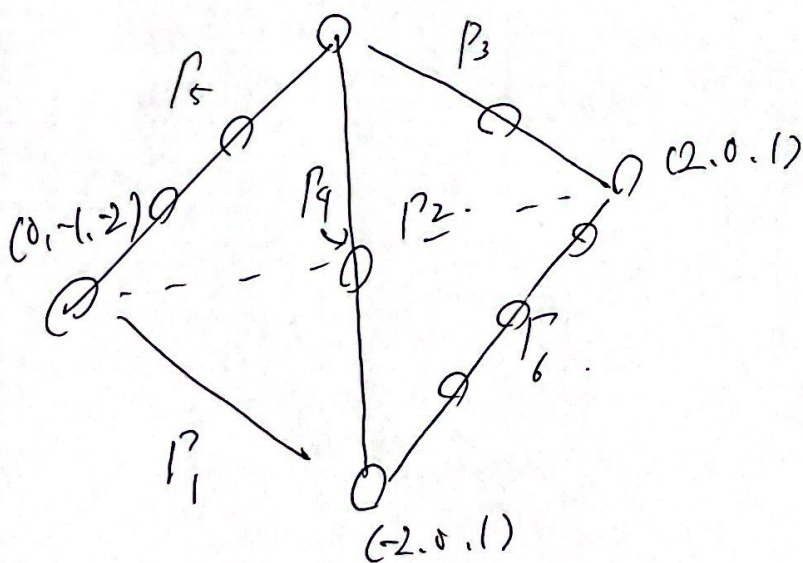
$f = W^3 + X^4 + Y^4 + Z^6$ (k3 singularity).

$w = (1/3, 1/4, 1/4, 1/6)$.

$N = \text{lcm}(3, 4, 6) = 12$.

$(q_1, q_2, q_3, q_4) = (4, 3, 3, 2)$.

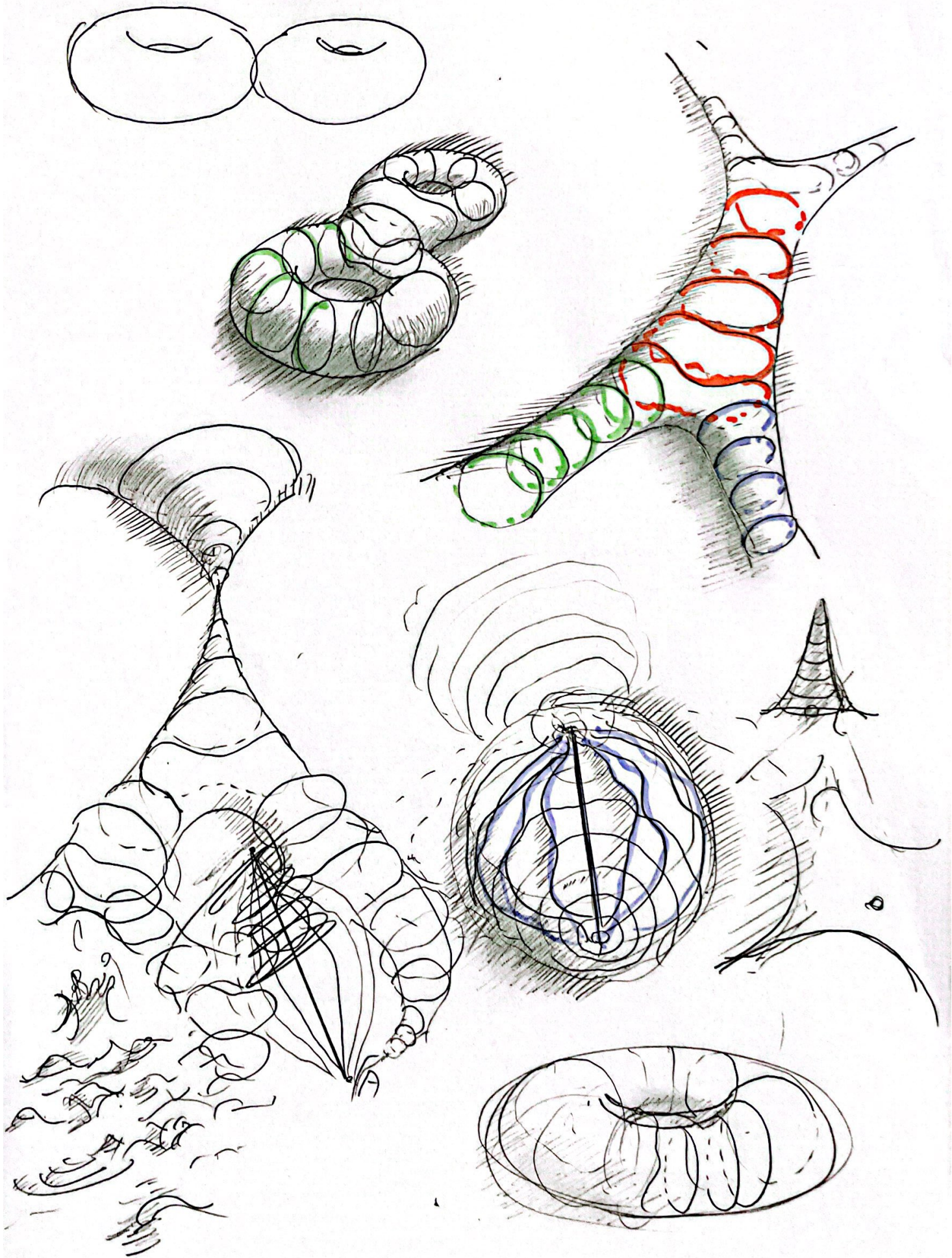
$(0, 2, 1)$.



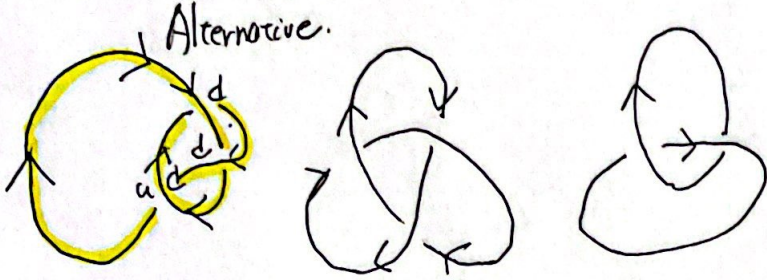
$U \oplus U(3) \oplus A_2 \oplus D_4$.

$sgn = (2, 4)$

10



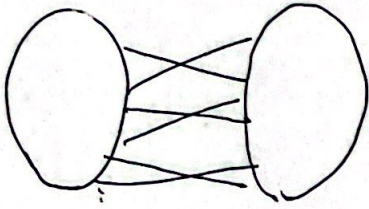
simple Towards know theory from random matrices.



$$Z = \int e^{g \left(\sum_{i=1}^N \phi_i^2 \right)^2 + \sum \phi_0^2}$$

$$\int dH e^{-g \text{tr} M^2} (\prod_j (\text{tr} M^j)^{2j})$$

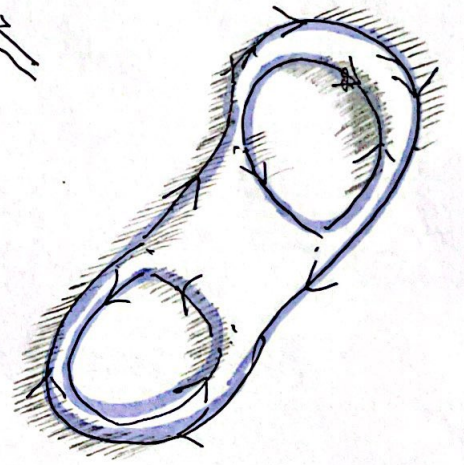
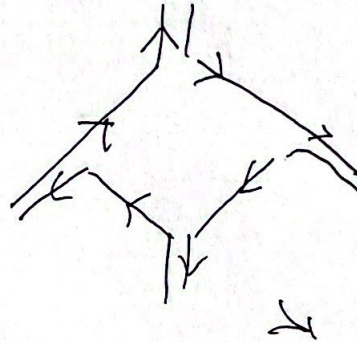
$$\langle (\text{tr} M^j)^2 \rangle \quad \text{tr}(M^3) = \text{tr}(M M M) \quad \sim \int \prod (\lambda_i - \lambda_j)^2 e^{-g \sum \lambda_i^2} f$$



Circle or knot.

Gaussian Average.

$$\langle \text{tr} M^4 \rangle = \langle \text{tr} \underbrace{M \cdot M}_4 \rangle$$

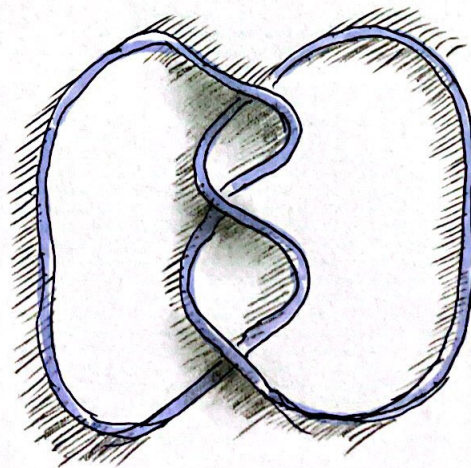
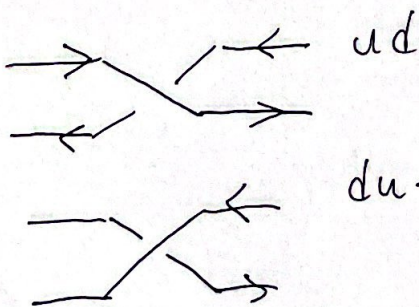


$$\lim_{N \rightarrow 0} \frac{1}{N} \langle \text{tr} M^4 \rangle$$

$$= \frac{1}{N} \langle \text{tr} \underbrace{M \cdot M}_4 \rangle$$

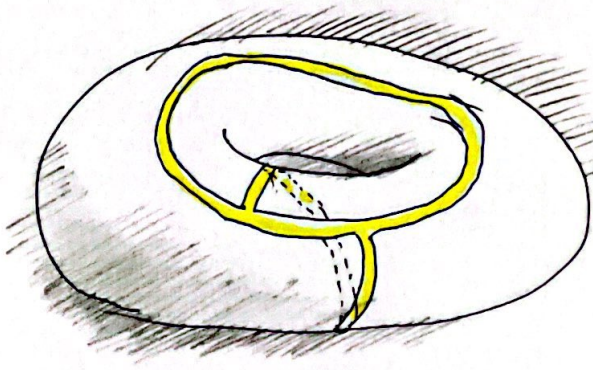
$$\sim (2N^3 + N) \frac{1}{N}$$

$$\sim 1.$$



$$Osp(n|2m)$$

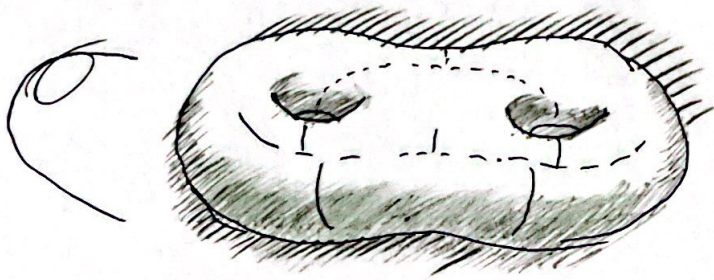
$$\lim_{\lambda \rightarrow \infty} SU(\infty|m).$$



Kontsevich.
ribbon graph.

$$\langle \text{Tr} M^3 \rangle = \frac{1}{24} \quad (g=1).$$

$$\langle \text{Tr} M^3 \rangle^2 = \frac{1}{2(24 \cdot 24)^2} \quad (g=2).$$



E. Brezin and SH

$$\int d\alpha_1 \dots \log \left(1 - \frac{\sigma_1}{\alpha} \right) \dots$$

$$\frac{2^k}{\chi^2} \prod_{i=1}^k \sinh \left(\frac{\chi \sigma_i}{2} \right) = f(\sigma_1, \dots, \sigma_k).$$

$$= \langle \text{tr} e^{\alpha_1 M} \dots \text{tr} e^{\alpha_k M} \rangle$$