

```

K<s5>:=QuadraticField(5);
a:=(1+s5)/2;
b:=(1-s5)/2;
G:=MatrixGroup<4,K|[0,1/2,b/2,-a/2,-1/2,0,-a/2,-b/2,-b/2,a/2,0,1/2,a/2,b/2,-1/2,0],
[1/2,1/2,1/2,-1/2,-1/2,1/2,-1/2,-1/2,-1/2,1/2,1/2,1/2,1/2,-1/2,1/2],[0,1,0,0,-1,0,0,0,0,0,0,1,0,0,-1,0],
[0,1,0,0,-1,0,0,0,0,0,-1,0,0,1,0],[1/2,1/2,1/2,1/2,1/2,-1/2,-1/2,1/2,-1/2,1/2,-1/2,1/2,-1/2,1/2,-1/2,1/2],
[1,0,0,0,0,1,0,0,0,0,1,0,0,0,-1],[0,1/2,b/2,a/2,-1/2,0,-a/2,b/2,-b/2,a/2,0,-1/2,-a/2,-b/2,1/2,0]>;
#G;
MaximalSubgroups(G);
IsSimple(G);
CT:=CharacterTable(G);
CT;

```

\*\*\*\*\*

14400  
Conjugacy classes of subgroups  
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[1] Order 144      Length 100
MatrixGroup(4, K) of order 2^4 * 3^2
Generators:
[ 0 1/4*(-s5 + 1) 1/4*(-s5 - 1) 1/2]
[ 1/4*(s5 - 1) 0 -1/2 1/4*(-s5 - 1)]
[ 1/4*(s5 + 1) 1/2 0 1/4*(s5 - 1)]
[ -1/2 1/4*(s5 + 1) 1/4*(-s5 + 1) 0]

[ 1/4*(s5 + 1) 1/4*(s5 - 1) -1/2 0]
[ 0 0 0 1]
[1/4*(-s5 + 1) -1/2 1/4*(-s5 - 1) 0]
[ 1/2 1/4*(-s5 - 1) 1/4*(s5 - 1) 0]

[1/4*(-s5 - 1) -1/2 0 1/4*(-s5 + 1)]
[ -1/2 1/4*(s5 + 1) 1/4*(-s5 + 1) 0]
[ 0 1/4*(-s5 + 1) 1/4*(-s5 - 1) 1/2]
[1/4*(-s5 + 1) 0 1/2 1/4*(s5 + 1)]

[ 1/2 1/4*(-s5 - 1) 1/4*(s5 - 1) 0]
[ 1/4*(s5 + 1) 1/2 0 1/4*(s5 - 1)]
[1/4*(-s5 + 1) 0 1/2 1/4*(s5 + 1)]
[ 0 1/4*(-s5 + 1) 1/4*(-s5 - 1) 1/2]

[ 1/2 1/4*(s5 - 1) 0 1/4*(-s5 - 1)]
[1/4*(-s5 + 1) 1/2 1/4*(-s5 - 1) 0]
[ 0 1/4*(s5 + 1) 1/2 1/4*(s5 - 1)]
[ 1/4*(s5 + 1) 0 1/4*(-s5 + 1) 1/2]

[-1 0 0 0]
[ 0 -1 0 0]
[ 0 0 -1 0]
[ 0 0 0 -1]

```

```

[2] Order 240      Length 60
MatrixGroup(4, K) of order 2^4 * 3 * 5
Generators:
[ -1/2 1/4*(s5 + 1) 1/4*(s5 - 1) 0]
[ 1/4*(s5 + 1) 1/4*(s5 - 1) 1/2 0]
[ 1/4*(s5 - 1) 1/2 1/4*(-s5 - 1) 0]
[ 0 0 0 -1]

[ 0 1/4*(s5 - 1) 1/4*(s5 + 1) -1/2]
[ -1/2 -1/2 1/2 1/2]
[ 1/4*(s5 - 1) 1/4*(-s5 - 1) 0 -1/2]
[1/4*(-s5 - 1) 0 1/4*(-s5 + 1) -1/2]

```

$$\begin{bmatrix} 1/2 & 1/4*(s5 - 1) & 0 & 1/4*(-s5 - 1) \\ 1/4*(s5 - 1) & 1/4*(s5 + 1) & 0 & 1/2 \\ 0 & 0 & 1 & 0 \\ 1/4*(-s5 - 1) & 1/2 & 0 & 1/4*(-s5 + 1) \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

- [3] Order 240      Length 60  
MatrixGroup(4, K) of order  $2^4 * 3 * 5$

Generators:

$$\begin{bmatrix} 1/4*(-s5 + 1) & -1/2 & 1/4*(s5 + 1) & 0 \\ -1/2 & 1/4*(s5 + 1) & 1/4*(s5 - 1) & 0 \\ 1/4*(s5 + 1) & 1/4*(s5 - 1) & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & 1/4*(-s5 + 1) & 0 & 1/4*(s5 + 1) \\ 1/4*(-s5 + 1) & -1/2 & 1/4*(-s5 - 1) & 0 \\ 0 & 1/4*(s5 + 1) & -1/2 & 1/4*(s5 - 1) \\ 1/4*(s5 + 1) & 0 & 1/4*(-s5 + 1) & -1/2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

- [4] Order 400      Length 36  
MatrixGroup(4, K) of order  $2^4 * 5^2$

Generators:

$$\begin{bmatrix} 0 & 1/4*(-s5 + 1) & 1/4*(-s5 - 1) & 1/2 \\ 1/4*(s5 - 1) & 0 & -1/2 & 1/4*(-s5 - 1) \\ 1/4*(s5 + 1) & 1/2 & 0 & 1/4*(s5 - 1) \\ -1/2 & 1/4*(s5 + 1) & 1/4*(-s5 + 1) & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1/2 & 1/4*(s5 - 1) & 1/4*(s5 + 1) \\ 0 & 1/4*(s5 - 1) & 1/4*(-s5 - 1) & 1/2 \\ 0 & 1/4*(s5 + 1) & 1/2 & 1/4*(s5 - 1) \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1/4*(s5 + 1) & -1/2 & 1/4*(s5 - 1) \\ 0 & -1/2 & 1/4*(-s5 + 1) & 1/4*(s5 + 1) \\ 0 & 1/4*(s5 - 1) & 1/4*(s5 + 1) & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1/4*(s5 + 1) & 0 & 1/4*(s5 - 1) & 1/2 \\ 0 & 1/4*(s5 + 1) & 1/2 & 1/4*(-s5 + 1) \\ 1/4*(-s5 + 1) & -1/2 & 1/4*(s5 + 1) & 0 \\ -1/2 & 1/4*(s5 - 1) & 0 & 1/4*(s5 + 1) \end{bmatrix}$$

$$\begin{bmatrix} 1/4*(s5 - 1) & 1/4*(-s5 - 1) & 0 & -1/2 \\ 1/4*(s5 + 1) & 1/4*(s5 - 1) & 1/2 & 0 \\ 0 & -1/2 & 1/4*(s5 - 1) & 1/4*(s5 + 1) \\ 1/2 & 0 & 1/4*(-s5 - 1) & 1/4*(s5 - 1) \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

- [5] Order 576      Length 25  
MatrixGroup(4, K) of order  $2^6 * 3^2$

Generators:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/4*(s5 + 1) & 1/2 & 1/4*(s5 - 1) \\ 0 & 1/2 & 1/4*(-s5 + 1) & 1/4*(-s5 - 1) \end{bmatrix}$$

[ 0 1/4\*(s5 - 1) 1/4\*(-s5 - 1) 1/2]

[ 1/2 -1/2 1/2 -1/2]

[ 1/2 1/2 -1/2 -1/2]

[-1/2 1/2 1/2 -1/2]

[ 1/2 1/2 1/2 1/2]

[ -1/2 1/4\*(s5 + 1) 1/4\*(s5 - 1) 0]

[1/4\*(-s5 - 1) -1/2 0 1/4\*(s5 - 1)]

[1/4\*(-s5 + 1) 0 -1/2 1/4\*(-s5 - 1)]

[ 0 1/4\*(-s5 + 1) 1/4\*(s5 + 1) -1/2]

[ 0 1/4\*(-s5 + 1) 1/4\*(-s5 - 1) -1/2]

[ 1/4\*(s5 - 1) 0 1/2 1/4\*(-s5 - 1)]

[ 1/4\*(s5 + 1) -1/2 0 1/4\*(s5 - 1)]

[ 1/2 1/4\*(s5 + 1) 1/4\*(-s5 + 1) 0]

[ 0 -1/2 1/4\*(-s5 + 1) 1/4\*(-s5 - 1)]

[ 1/2 0 1/4\*(-s5 - 1) 1/4\*(s5 - 1)]

[ 1/4\*(s5 - 1) 1/4\*(s5 + 1) 0 -1/2]

[ 1/4\*(s5 + 1) 1/4\*(-s5 + 1) 1/2 0]

[ 0 1/4\*(-s5 - 1) 1/2 1/4\*(s5 - 1)]

[ 1/4\*(s5 + 1) 0 1/4\*(s5 - 1) -1/2]

[ -1/2 1/4\*(-s5 + 1) 0 1/4\*(-s5 - 1)]

[1/4\*(-s5 + 1) 1/2 1/4\*(s5 + 1) 0]

[ 0 -1/2 1/4\*(-s5 + 1) 1/4\*(s5 + 1)]

[ 1/2 0 1/4\*(-s5 - 1) 1/4\*(-s5 + 1)]

[ 1/4\*(s5 - 1) 1/4\*(s5 + 1) 0 1/2]

[1/4\*(-s5 - 1) 1/4\*(s5 - 1) -1/2 0]

[-1 0 0 0]

[0 -1 0 0]

[0 0 -1 0]

[0 0 0 -1]

[6] Order 7200 Length 1  
MatrixGroup(4, K) of order 2^5 \* 3^2 \* 5^2

Generators:

[ 1/4\*(s5 - 1) 1/4\*(-s5 - 1) 0 1/2]

[1/4\*(-s5 - 1) -1/2 0 1/4\*(-s5 + 1)]

[ 0 0 1 0]

[ 1/2 1/4\*(-s5 + 1) 0 1/4\*(-s5 - 1)]

[ 1 0 0 0]

[ 0 1/4\*(-s5 + 1) 1/4\*(-s5 - 1) -1/2]

[ 0 1/4\*(-s5 - 1) 1/2 1/4\*(-s5 + 1)]

[ 0 1/2 1/4\*(s5 - 1) 1/4\*(-s5 - 1)]

[-1 0 0 0]

[0 -1 0 0]

[0 0 -1 0]

[0 0 0 -1]

false

### Character Table of Group G

-----

Class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Size	1	1	60	60	450	40	400	60	1800	24	24	144	144	288	40	400	1200
Order	1	2	2	2	2	3	3	4	4	5	5	5	5	5	6	6	6

-----  
p = 2 1 1 1 1 1 6 7 2 5 11 10 13 12 14 6 7 7  
p = 3 1 2 3 4 5 1 1 8 9 11 10 13 12 14 2 2 4  
p = 5 1 2 3 4 5 6 7 8 9 1 1 1 1 1 15 16 17  
-----

X.1 + 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
X.2 + 1 1 -1 -1 1 1 1 1 -1 1 1 1 1 1 1 1 -1  
X.3 + 4 -4 2 -2 0 -2 1 0 0 Z1 Z1#2 Z8 Z8#2 -1 2 -1 1  
X.4 + 4 -4 -2 2 0 -2 1 0 0 Z1#2 Z1 Z8#2 Z8 -1 2 -1 -1  
X.5 + 4 -4 -2 2 0 -2 1 0 0 Z1 Z1#2 Z8 Z8#2 -1 2 -1 -1  
X.6 + 4 -4 2 -2 0 -2 1 0 0 Z1#2 Z1 Z8#2 Z8 -1 2 -1 1  
X.7 + 6 6 0 0 -2 3 0 2 0 Z2 Z2#2 -Z1-Z1#2 1 3 0 0  
X.8 + 6 6 0 0 -2 3 0 2 0 Z2#2 Z2-Z1#2 -Z1 1 3 0 0  
X.9 + 8 8 0 0 0 5 2 4 0 3 3 -2 -2 -2 5 2 0  
X.10 + 8 -8 0 0 0 -4 2 0 0 -2 -2 -2 -2 3 4 -2 0  
X.11 + 9 9 3 3 1 0 0 -3 -1 Z3 Z3#2 Z8 Z8#2 -1 0 0 0  
X.12 + 9 9 3 3 1 0 0 -3 -1 Z3#2 Z3 Z8#2 Z8 -1 0 0 0  
X.13 + 9 9 -3 -3 1 0 0 -3 1 Z3 Z3#2 Z8 Z8#2 -1 0 0 0  
X.14 + 9 9 -3 -3 1 0 0 -3 1 Z3#2 Z3 Z8#2 Z8 -1 0 0 0  
X.15 + 10 10 0 0 2 4 -2 6 0 5 5 0 0 0 4 -2 0  
X.16 + 16 -16 4 -4 0 4 1 0 0 -4 -4 1 1 1 -4 -1 -1  
X.17 + 16 16 4 4 0 4 1 0 0 -4 -4 1 1 1 4 1 1  
X.18 + 16 -16 -4 4 0 4 1 0 0 -4 -4 1 1 1 -4 -1 1  
X.19 + 16 16 -4 -4 0 4 1 0 0 -4 -4 1 1 1 4 1 -1  
X.20 + 16 -16 0 0 0 -2 -2 0 0 Z4 Z4#2 -Z1-Z1#2 1 2 2 0  
X.21 + 16 -16 0 0 0 -2 -2 0 0 Z4#2 Z4-Z1#2 -Z1 1 2 2 0  
X.22 + 18 18 0 0 2 0 0 -6 0 3 3 -2 -2 3 0 0 0  
X.23 + 24 24 0 0 0 3 0 -4 0 Z5 Z5#2 Z1 Z1#2 -1 3 0 0  
X.24 + 24 24 0 0 0 3 0 -4 0 Z5#2 Z5 Z1#2 Z1 -1 3 0 0  
X.25 + 24 -24 0 0 0 -6 0 0 0 Z6 Z6#2 Z1 Z1#2 -1 6 0 0  
X.26 + 24 -24 0 0 0 -6 0 0 0 Z6#2 Z6 Z1#2 Z1 -1 6 0 0  
X.27 + 25 25 5 5 1 -5 1 5 1 0 0 0 0 0 0 -5 1 -1  
X.28 + 25 25 -5 -5 1 -5 1 5 -1 0 0 0 0 0 0 -5 1 1  
X.29 + 30 30 0 0 -2 -3 0 -2 0 Z7 Z7#2 0 0 0 -3 0 0  
X.30 + 30 30 0 0 -2 -3 0 -2 0 Z7#2 Z7 0 0 0 -3 0 0  
X.31 + 36 -36 -6 6 0 0 0 0 0 6 6 1 1 1 0 0 0  
X.32 + 36 -36 6 -6 0 0 0 0 0 6 6 1 1 1 0 0 0  
X.33 + 40 40 0 0 0 1 -2 4 0 -5 -5 0 0 0 1 -2 0  
X.34 + 48 -48 0 0 0 6 0 0 0 -2 -2 -2 -2 -2 -6 0 0

-----  
Class | 18 19 20 21 22 23 24 25 26 27 28 29 30 31  
Size | 1200 24 24 144 144 288 720 720 720 720 1200 480 480 720  
Order | 6 10 10 10 10 10 10 10 10 10 10 12 15 15 20  
-----

p = 2 7 10 11 12 13 14 13 13 12 12 15 30 29 20  
p = 3 3 20 19 22 21 23 26 27 24 25 8 11 10 32  
p = 5 18 2 2 2 2 2 4 3 4 3 28 6 6 8  
-----

X.1 + 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
X.2 + -1 1 1 1 1 1 1 -1 -1 -1 -1 1 1 1 1  
X.3 + -1-Z1#2 -Z1-Z8#2 -Z8 1 Z9 -Z9 Z9#2-Z9#2 0 Z9 Z9#2 0  
X.4 + 1 -Z1-Z1#2 -Z8-Z8#2 1-Z9#2 Z9#2 -Z9 Z9 0 Z9#2 Z9 0  
X.5 + 1-Z1#2 -Z1-Z8#2 -Z8 1 -Z9 Z9-Z9#2 Z9#2 0 Z9 Z9#2 0  
X.6 + -1 -Z1-Z1#2 -Z8-Z8#2 1 Z9#2-Z9#2 Z9 -Z9 0 Z9#2 Z9 0  
X.7 + 0 Z2#2 Z2-Z1#2 -Z1 1 0 0 0 0 -1 Z9#2 Z9-Z9#2  
X.8 + 0 Z2 Z2#2 -Z1-Z1#2 1 0 0 0 0 -1 Z9 Z9#2 -Z9  
X.9 + 0 3 3 -2 -2 -2 0 0 0 0 1 0 0 -1  
X.10 + 0 2 2 2 2 -3 0 0 0 0 0 1 1 0  
X.11 + 0 Z3#2 Z3 Z8#2 Z8 -1 Z9 Z9 Z9#2 Z9#2 0 0 0 -Z9#2  
X.12 + 0 Z3 Z3#2 Z8 Z8#2 -1 Z9#2 Z9#2 Z9 Z9 0 0 0 -Z9  
X.13 + 0 Z3#2 Z3 Z8#2 Z8 -1 -Z9 -Z9-Z9#2-Z9#2 0 0 0 -Z9#2  
X.14 + 0 Z3 Z3#2 Z8 Z8#2 -1-Z9#2-Z9#2 -Z9 -Z9 0 0 0 -Z9

```

X.15 + 0 5 5 0 0 0 0 0 0 0 0 0 -1 -1 1
X.16 + 1 4 4 -1 -1 -1 1 -1 1 -1 0 -1 -1 0
X.17 + 1 -4 -4 1 1 1 -1 -1 -1 -1 0 -1 -1 0
X.18 + -1 4 4 -1 -1 -1 -1 1 -1 1 0 -1 -1 0
X.19 + -1 -4 -4 1 1 1 1 1 1 1 0 -1 -1 0
X.20 + 0-Z4#2 -Z4 Z1#2 Z1 -1 0 0 0 0 0 Z9 Z9#2 0
X.21 + 0 -Z4-Z4#2 Z1 Z1#2 -1 0 0 0 0 0 Z9#2 Z9 0
X.22 + 0 3 3 -2 -2 3 0 0 0 0 0 0 0 -1
X.23 + 0 Z5#2 Z5 Z1#2 Z1 -1 0 0 0 0 -1 Z9#2 Z9 1
X.24 + 0 Z5 Z5#2 Z1 Z1#2 -1 0 0 0 0 -1 Z9 Z9#2 1
X.25 + 0-Z6#2 -Z6-Z1#2 -Z1 1 0 0 0 0 0 -1 -1 0
X.26 + 0 -Z6-Z6#2 -Z1-Z1#2 1 0 0 0 0 0 -1 -1 0
X.27 + -1 0 0 0 0 0 0 0 0 0 -1 0 0 0
X.28 + 1 0 0 0 0 0 0 0 0 0 -1 0 0 0
X.29 + 0 Z7#2 Z7 0 0 0 0 0 0 0 1-Z9#2 -Z9 Z9
X.30 + 0 Z7 Z7#2 0 0 0 0 0 0 0 1 -Z9-Z9#2 Z9#2
X.31 + 0 -6 -6 -1 -1 -1 1 -1 1 -1 0 0 0 0
X.32 + 0 -6 -6 -1 -1 -1 -1 1 -1 1 0 0 0 0
X.33 + 0 -5 -5 0 0 0 0 0 0 0 1 1 1 -1
X.34 + 0 2 2 2 2 2 0 0 0 0 0 1 1 0

```

```

-----
Class | 32 33 34
Size | 720 480 480
Order | 20 30 30

```

```

-----
p = 2 19 29 30
p = 3 31 20 19
p = 5 8 15 15

```

```

-----
X.1 + 1 1 1
X.2 + 1 1 1
X.3 + 0-Z9#2 -Z9
X.4 + 0 -Z9-Z9#2
X.5 + 0-Z9#2 -Z9
X.6 + 0 -Z9-Z9#2
X.7 + -Z9 Z9 Z9#2
X.8 + -Z9#2 Z9#2 Z9
X.9 + -1 0 0
X.10 + 0 -1 -1
X.11 + -Z9 0 0
X.12 + -Z9#2 0 0
X.13 + -Z9 0 0
X.14 + -Z9#2 0 0
X.15 + 1 -1 -1
X.16 + 0 1 1
X.17 + 0 -1 -1
X.18 + 0 1 1
X.19 + 0 -1 -1
X.20 + 0-Z9#2 -Z9
X.21 + 0 -Z9-Z9#2
X.22 + -1 0 0
X.23 + 1 Z9 Z9#2
X.24 + 1 Z9#2 Z9
X.25 + 0 1 1
X.26 + 0 1 1
X.27 + 0 0 0
X.28 + 0 0 0
X.29 + Z9#2 -Z9-Z9#2
X.30 + Z9-Z9#2 -Z9
X.31 + 0 0 0
X.32 + 0 0 0
X.33 + -1 1 1

```

X.34 + 0 -1 -1

#### Explanation of Character Value Symbols

-----

# denotes algebraic conjugation, that is,  
#k indicates replacing the root of unity  $w$  by  $w^k$

Z1 = (CyclotomicField(5: Sparse := true)) ! [ RationalField() | -2, 0, -2, -2 ]

Z2 = (CyclotomicField(5: Sparse := true)) ! [ RationalField() | 3, 0, -1, -1 ]

Z3 = (CyclotomicField(5: Sparse := true)) ! [ RationalField() | 3, 0, 3, 3 ]

Z4 = (CyclotomicField(5: Sparse := true)) ! [ RationalField() | -2, 0, 4, 4 ]

Z5 = (CyclotomicField(5: Sparse := true)) ! [ RationalField() | -3, 0, -4, -4 ]

Z6 = (CyclotomicField(5: Sparse := true)) ! [ RationalField() | 2, 0, 6, 6 ]

Z7 = (CyclotomicField(5: Sparse := true)) ! [ RationalField() | 0, 0, -5, -5 ]

Z8 = (CyclotomicField(5: Sparse := true)) ! [ RationalField() | 1, 0, -1, -1 ]

Z9 = (CyclotomicField(5: Sparse := true)) ! [ RationalField() | 1, 0, 1, 1 ]