

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ & 2 & & \\ & & 2 & \\ & & & 2 \end{pmatrix}$$

F4 Lattice

\*Kissing Number; 24

\*The group order: 1152

\*The shortest Vectors

\*\*\*\*\*

```

[[-2 0 0 0],
 [-1 -1 -1 -1],
 [-1 -1 -1 1],
 [-1 -1 1 -1],
 [-1 -1 1 1],
 [-1 1 -1 -1],
 [-1 1 -1 1],
 [-1 1 1 -1],
 [-1 1 1 1],
 [ 0 -2 0 0],
 [ 0 0 -2 0],
 [ 0 0 0 -2],
 [0 0 0 2],
 [0 0 2 0],
 [0 2 0 0],
 [ 1 -1 -1 -1],
 [ 1 -1 -1 1],
 [ 1 -1 1 -1],
 [ 1 -1 1 1],
 [ 1 1 -1 -1],
 [ 1 1 -1 1],
 [ 1 1 1 -1],
 [1 1 1 1],
 [2 0 0 0]]

```

\*\*\*\*\*

The equilateral triangle: 224

2\*sqrt(3) : 96

4\*sqrt(3) : 96

6\*sqrt(3) : 32

\*\*\*\*\*

```

[[[-2 0 0 0], [-1 -1 -1 -1], [-1 -1 -1 1]],
 [[-2 0 0 0], [-1 -1 -1 -1], [-1 -1 1 -1]],
 [[-2 0 0 0], [-1 -1 -1 -1], [-1 1 -1 -1]],
 [[-2 0 0 0], [-1 -1 -1 1], [-1 -1 1 1]],
 [[-2 0 0 0], [-1 -1 -1 1], [-1 1 -1 1]],
 [[-2 0 0 0], [-1 -1 1 -1], [-1 -1 1 1]],
 [[-2 0 0 0], [-1 -1 1 -1], [-1 1 1 -1]],
 [[-2 0 0 0], [-1 -1 1 1], [-1 1 1 1]],
 [[-2 0 0 0], [-1 1 -1 -1], [-1 1 -1 1]],
 [[-2 0 0 0], [-1 1 -1 -1], [-1 1 1 -1]],
 [[-2 0 0 0], [-1 1 -1 1], [-1 1 1 1]],
 [[-2 0 0 0], [-1 1 1 -1], [-1 1 1 1]],
 [[-2 0 0 0], [-1 1 1 -1], [-1 1 1 1]],
 [[-2 0 0 0], [ 0 -2 0 0], [ 0 0 -2 0]],
 [[-2 0 0 0], [ 0 -2 0 0], [ 0 0 0 -2]],
 [[-2 0 0 0], [ 0 -2 0 0], [0 0 0 2]],
 [[-2 0 0 0], [ 0 -2 0 0], [0 0 2 0]],
 [[-2 0 0 0], [ 0 0 -2 0], [ 0 0 0 -2]],
 [[-2 0 0 0], [ 0 0 -2 0], [0 0 0 2]],
 [[-2 0 0 0], [ 0 0 -2 0], [0 2 0 0]],
 [[-2 0 0 0], [ 0 0 0 -2], [0 0 2 0]],
 [[-2 0 0 0], [ 0 0 0 -2], [0 2 0 0]],
 [[-2 0 0 0], [0 0 0 2], [0 0 2 0]],
 [[-2 0 0 0], [0 0 0 2], [0 2 0 0]],

```



$[-1 -1 1 -1], [1 -1 1 1], [1 1 1 -1],$   
 $[-1 -1 1 1], [-1 1 -1 -1], [2 0 0 0],$   
 $[-1 -1 1 1], [-1 1 -1 1], [-1 1 1 -1],$   
 $[-1 -1 1 1], [-1 1 -1 1], [1 -1 -1 1],$   
 $[-1 -1 1 1], [-1 1 -1 1], [1 1 1 1],$   
 $[-1 -1 1 1], [-1 1 1 -1], [1 -1 1 -1],$   
 $[-1 -1 1 1], [-1 1 1 -1], [1 1 1 1],$   
 $[-1 -1 1 1], [-1 1 1 1], [0 0 0 2],$   
 $[-1 -1 1 1], [-1 1 1 1], [0 0 2 0],$   
 $[-1 -1 1 1], [0 -2 0 0], [1 -1 1 1],$   
 $[-1 -1 1 1], [0 0 -2 0], [1 1 1 -1],$   
 $[-1 -1 1 1], [0 0 0 -2], [1 1 -1 1],$   
 $[-1 -1 1 1], [0 0 0 2], [1 -1 1 1],$   
 $[-1 -1 1 1], [0 0 2 0], [1 -1 1 1],$   
 $[-1 -1 1 1], [0 2 0 0], [1 -1 -1 -1],$   
 $[-1 -1 1 1], [1 -1 -1 1], [1 -1 1 -1],$   
 $[-1 -1 1 1], [1 -1 -1 1], [1 1 1 1],$   
 $[-1 -1 1 1], [1 -1 1 -1], [1 1 1 1],$   
 $[-1 1 -1 -1], [-1 1 -1 1], [0 0 -2 0],$   
 $[-1 1 -1 -1], [-1 1 -1 1], [0 2 0 0],$   
 $[-1 1 -1 -1], [-1 1 1 -1], [0 0 0 -2],$   
 $[-1 1 -1 -1], [-1 1 1 -1], [0 2 0 0],$   
 $[-1 1 -1 -1], [-1 1 1 1], [1 1 -1 1],$   
 $[-1 1 -1 -1], [-1 1 1 1], [1 1 1 -1],$   
 $[-1 1 -1 -1], [0 -2 0 0], [1 1 1 1],$   
 $[-1 1 -1 -1], [0 0 -2 0], [1 1 -1 -1],$   
 $[-1 1 -1 -1], [0 0 0 -2], [1 1 -1 -1],$   
 $[-1 1 -1 -1], [0 0 0 2], [1 -1 1 -1],$   
 $[-1 1 -1 -1], [0 0 2 0], [1 -1 -1 1],$   
 $[-1 1 -1 -1], [0 2 0 0], [1 1 -1 -1],$   
 $[-1 1 -1 -1], [1 -1 -1 -1], [1 1 -1 1],$   
 $[-1 1 -1 -1], [1 -1 -1 -1], [1 1 1 -1],$   
 $[-1 1 -1 -1], [1 1 -1 1], [1 1 1 -1],$   
 $[-1 1 -1 1], [-1 1 1 -1], [1 1 -1 -1],$   
 $[-1 1 -1 1], [-1 1 1 -1], [1 1 1 1],$   
 $[-1 1 -1 1], [-1 1 1 1], [0 0 0 2],$   
 $[-1 1 -1 1], [-1 1 1 1], [0 2 0 0],$   
 $[-1 1 -1 1], [0 -2 0 0], [1 1 1 -1],$   
 $[-1 1 -1 1], [0 0 -2 0], [1 1 -1 1],$   
 $[-1 1 -1 1], [0 0 0 -2], [1 -1 1 1],$   
 $[-1 1 -1 1], [0 0 0 2], [1 1 -1 1],$   
 $[-1 1 -1 1], [0 0 2 0], [1 -1 -1 -1],$   
 $[-1 1 -1 1], [0 2 0 0], [1 1 -1 1],$   
 $[-1 1 -1 1], [1 -1 -1 1], [1 1 -1 -1],$   
 $[-1 1 -1 1], [1 -1 -1 1], [1 1 1 1],$   
 $[-1 1 -1 1], [1 1 -1 -1], [1 1 1 1],$   
 $[-1 1 1 -1], [-1 1 1 1], [0 0 2 0],$   
 $[-1 1 1 -1], [-1 1 1 1], [0 2 0 0],$   
 $[-1 1 1 -1], [0 -2 0 0], [1 1 -1 1],$   
 $[-1 1 1 -1], [0 0 -2 0], [1 -1 1 1],$   
 $[-1 1 1 -1], [0 0 0 -2], [1 1 1 -1],$   
 $[-1 1 1 -1], [0 0 0 2], [1 -1 -1 -1],$   
 $[-1 1 1 -1], [0 0 2 0], [1 1 1 -1],$   
 $[-1 1 1 -1], [0 2 0 0], [1 1 1 -1],$   
 $[-1 1 1 -1], [1 -1 1 -1], [1 1 -1 -1],$   
 $[-1 1 1 -1], [1 -1 1 -1], [1 1 1 1],$   
 $[-1 1 1 -1], [1 1 -1 -1], [1 1 1 1],$   
 $[-1 1 1 1], [0 -2 0 0], [1 1 -1 -1],$   
 $[-1 1 1 1], [0 0 -2 0], [1 -1 1 -1],$   
 $[-1 1 1 1], [0 0 0 -2], [1 -1 -1 1],$   
 $[-1 1 1 1], [0 0 0 2], [1 1 1 1],$   
 $[-1 1 1 1], [0 0 2 0], [1 1 1 1],$   
 $[-1 1 1 1], [0 2 0 0], [1 1 1 1],$   
 $[-1 1 1 1], [1 -1 1 1], [1 1 -1 1],$   
 $[-1 1 1 1], [1 -1 1 1], [1 1 1 -1],$   
 $[-1 1 1 1], [1 1 -1 1], [1 1 1 -1],$   
 $[0 -2 0 0], [0 0 -2 0], [0 0 0 -2],$   
 $[0 -2 0 0], [0 0 -2 0], [0 0 0 2],$   
 $[0 -2 0 0], [0 0 -2 0], [2 0 0 0],$   
 $[0 -2 0 0], [0 0 0 -2], [0 0 2 0],$   
 $[0 -2 0 0], [0 0 0 -2], [2 0 0 0],$

```

[[ 0 -2 0 0], [0 0 0 2], [0 0 2 0]],
[[ 0 -2 0 0], [0 0 0 2], [2 0 0 0]],
[[ 0 -2 0 0], [0 0 2 0], [2 0 0 0]],
[[ 0 -2 0 0], [1 -1 -1 -1], [1 -1 -1 1]],
[[ 0 -2 0 0], [1 -1 -1 -1], [1 -1 1 -1]],
[[ 0 -2 0 0], [1 -1 -1 1], [1 -1 1 1]],
[[ 0 -2 0 0], [1 -1 1 -1], [1 -1 1 1]],
[[ 0 0 -2 0], [0 0 0 -2], [0 2 0 0]],
[[ 0 0 -2 0], [0 0 0 -2], [2 0 0 0]],
[[ 0 0 -2 0], [0 0 0 2], [0 2 0 0]],
[[ 0 0 -2 0], [0 0 0 2], [2 0 0 0]],
[[ 0 0 -2 0], [0 2 0 0], [2 0 0 0]],
[[ 0 0 -2 0], [1 -1 -1 -1], [1 -1 -1 1]],
[[ 0 0 -2 0], [1 -1 -1 -1], [1 1 -1 -1]],
[[ 0 0 -2 0], [1 -1 -1 1], [1 1 -1 1]],
[[ 0 0 -2 0], [1 1 -1 -1], [1 1 -1 1]],
[[ 0 0 0 -2], [0 0 2 0], [0 2 0 0]],
[[ 0 0 0 -2], [0 0 2 0], [2 0 0 0]],
[[ 0 0 0 -2], [0 2 0 0], [2 0 0 0]],
[[ 0 0 0 -2], [1 -1 -1 -1], [1 -1 1 -1]],
[[ 0 0 0 -2], [1 -1 -1 -1], [1 1 -1 -1]],
[[ 0 0 0 -2], [1 -1 1 -1], [1 1 1 -1]],
[[ 0 0 0 -2], [1 1 -1 -1], [1 1 1 -1]],
[[0 0 0 2], [0 0 2 0], [0 2 0 0]],
[[0 0 0 2], [0 0 2 0], [2 0 0 0]],
[[0 0 0 2], [0 2 0 0], [2 0 0 0]],
[[0 0 0 2], [1 -1 -1 1], [1 -1 1 1]],
[[0 0 0 2], [1 -1 -1 1], [1 1 -1 1]],
[[0 0 0 2], [1 -1 1 1], [1 1 1 1]],
[[0 0 0 2], [1 1 -1 1], [1 1 1 1]],
[[0 2 0 0], [0 2 0 0], [2 0 0 0]],
[[0 2 0 0], [1 -1 1 -1], [1 -1 1 1]],
[[0 2 0 0], [1 -1 1 -1], [1 1 1 -1]],
[[0 2 0 0], [1 -1 1 1], [1 1 1 1]],
[[0 2 0 0], [1 1 -1 -1], [1 1 1 1]],
[[0 2 0 0], [1 1 -1 -1], [1 1 1 -1]],
[[0 2 0 0], [1 1 -1 1], [1 1 1 1]],
[[0 2 0 0], [1 1 1 -1], [1 1 1 1]],
[[ 1 -1 -1 -1], [1 -1 -1 1], [2 0 0 0]],
[[ 1 -1 -1 -1], [1 -1 1 -1], [2 0 0 0]],
[[ 1 -1 -1 -1], [1 -1 1 1], [1 1 -1 1]],
[[ 1 -1 -1 -1], [1 -1 1 1], [1 1 1 -1]],
[[ 1 -1 -1 -1], [1 1 -1 -1], [2 0 0 0]],
[[ 1 -1 -1 -1], [1 1 -1 1], [1 1 1 -1]],
[[ 1 -1 -1 1], [1 -1 1 -1], [1 1 1 1]],
[[ 1 -1 -1 1], [1 -1 1 1], [2 0 0 0]],
[[ 1 -1 -1 1], [1 1 -1 -1], [1 1 1 1]],
[[ 1 -1 -1 1], [1 1 -1 1], [2 0 0 0]],
[[ 1 -1 1 -1], [1 -1 1 1], [2 0 0 0]],
[[ 1 -1 1 -1], [1 1 -1 -1], [1 1 1 1]],
[[ 1 -1 1 -1], [1 1 1 -1], [2 0 0 0]],
[[ 1 -1 1 1], [1 1 -1 1], [1 1 1 -1]],
[[ 1 -1 1 1], [1 1 1 1], [2 0 0 0]],
[[ 1 1 -1 -1], [1 1 -1 1], [2 0 0 0]],
[[ 1 1 -1 -1], [1 1 1 -1], [2 0 0 0]],
[[ 1 1 -1 1], [1 1 1 1], [2 0 0 0]],
[[ 1 1 1 -1], [1 1 1 1], [2 0 0 0]]

```

\*\*\*\*\*

The regular tetrahedron : 8

\*\*\*\*\*

```

[[[ 0 -2 0 0], [0 0 -2 0], [0 0 0 -2], [2 0 0 0]],
[[ 0 -2 0 0], [0 0 -2 0], [0 0 0 2], [2 0 0 0]],
[[ 0 -2 0 0], [0 0 0 -2], [0 0 2 0], [2 0 0 0]],
[[ 0 -2 0 0], [0 0 0 2], [0 0 2 0], [2 0 0 0]],
[[ 0 0 -2 0], [0 0 0 -2], [0 2 0 0], [2 0 0 0]],
[[ 0 0 -2 0], [0 0 0 2], [0 2 0 0], [2 0 0 0]],
[[ 0 0 0 -2], [0 0 2 0], [0 2 0 0], [2 0 0 0]]

```

```
[[0 0 0 2], [0 0 2 0], [0 2 0 0], [2 0 0 0]]
```

```
*****
```

```
8-point set for each vertex: 24
```

```
*****
```

```

[[0, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0],
[1, 1, 0, 0, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0],
[1, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0],
[1, 0, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0],
[1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0],
[1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0],
[1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1, 0],
[1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 0],
[0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0],
[0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1],
[0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0],
[0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0],
[0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1],
[0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1],
[0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 1, 0],
[0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1],
[0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0],
[0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 1],
[0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 1],
[0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1],
[0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0, 1, 1, 1],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]]

```

```
:
```

```

[[ 1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1
[1 1 1 1 1 1 1 1]]

```

```
*****
```

```
The code generated by the above.
```

```

1 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 1 1 1 0 0 0 1 1 0 1 0
0 0 1 0 0 0 0 0 0 1 1 0 1 0 0 1 1 0 1 0
0 0 0 1 0 0 0 0 0 1 0 1 0 1 0 0 1 1 0 1
0 0 0 0 1 0 0 0 0 1 0 0 1 1 0 0 1 1 0 0
0 0 0 0 0 1 0 0 0 1 1 0 0 1 0 1 1 1 1 0
0 0 0 0 0 0 1 0 0 1 0 1 0 1 0 1 0 0 1 0
0 0 0 0 0 0 0 1 0 0 0 1 0 1 1 0 0 1 0 0
0 0 0 0 0 0 0 0 1 0 0 0 1 1 1 0 0 0 1 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1

```

```

*****
[24, 10, 6] Linear Code over GF(2)
Permutation group G acting on a set of cardinality 24
Order = 18432 = 2^11 * 3^2
G = PermutationGroup([[(1, 15, 24, 10),(2, 4),(3, 20),(5, 22),(6, 7, 19, 18),(8, 9, 17,
16),(21, 23)],
[(1, 24),(2, 23),(3, 22),(6, 19),(7, 18),(11, 14)],
[(1, 21, 17, 13, 6, 2),(3, 20, 14, 22, 5, 11),(4, 8, 12, 19, 23, 24),(7, 15,
16,18, 10, 9)],
[(1, 6),(2, 7),(3, 11),(4, 10),(8, 17),(9, 13),(12, 16),(14, 22),(15, 21),(18,
23),(19,24)],
[(1, 24),(3, 22),(4, 21),(7, 18),(8, 17),(10, 15)],
[(1, 2),(3, 4),(5, 10),(6, 19),(7, 13),(8, 14),(11, 17),(12, 18),(15, 20),(21,
22),(23,24)]]
*****

```