Acylindrically hyperbolic groups.

- Intro
- Artin 11/10
- Recent 2-①
- Future.

Ex ・non-elementary hyperbolic groups

・mapping class groups of  unless $g=0$ $p\leq 3$. (Bowditch, 2008)

・$Out(F_n)$ $n\geq 2$ (Bestvina-Feighn 2010).

Non-Ex

- Direct products of two infinite groups. } Dahmani - Guirardel & Osin 2017
- Groups with infinite centers.

Def $G \overset{\text{isom}}{\curvearrowright} (X,d)$ geodesic space. This action is

acylindrical $\Leftrightarrow$ $\forall \varepsilon \geq 0$, $\exists N, R \geq 0$, s.t. if $d(x_1,x_2) \geq R$

$$\#\{g\in G \mid d(x_1, gx_1) \leq \varepsilon,\ d(x_2, gx_2) \leq \varepsilon\} \leq N.$$

Def An acylindrically hyperbolic group is a group acting non-elementarily and acylindrically on a hyp. sp.

2. Artin Groups.

Def Γ: a finite simple graph with edge labels $\in \mathbb{N}_{\geq 3} \cup \{\infty\}$.

$(\overset{v_1}{\circ} \overset{m}{\text{—}} \overset{v_2}{\circ} \in E(\Gamma))$ ← Label

$V(\Gamma)$: the vertex set $E(\Gamma)$: the edge set.

Artin grp } $A_\Gamma := \langle V(\Gamma) \mid \underbrace{v_1v_2v_1\cdots}_{m} = \underbrace{v_2v_1v_2\cdots}_{m},\ v_1v_2 = v_2v_1 \rangle$

Def. $W_\Gamma = \langle V(\Gamma) \mid \text{relations of } A_\Gamma,\ v^2 = 1 \ (v \in V(\Gamma)) \rangle$

↖ Coxeter group associated to Γ

Def

- A_Γ is of spherical (finite) type
  $\Leftrightarrow W_\Gamma$: finite.

- A_Γ is of infinite type $\Leftrightarrow$ not spherical

Conj (Charney & Morris-Wright, 2017).

All irreducible Artin groups of infinite type are acylindrically hyperbolic.

(i.e. Γ: connected.)

(任意の無限群のアルティン群は acylindrically hyperbolic).

Remark 1

All spherical Artin groups are acylindrically hyperbolic modulo centers. (Calvez-Wiest, 2017)

Remark 2

A_Γ: reducible $\Rightarrow$ A_Γ : directly indecomposable

$\Rightarrow$ Not acylindrically hyp.

3. Recent Results.

Notation.

$\Gamma^{<\infty}_{2}$:= a graph from Γ by

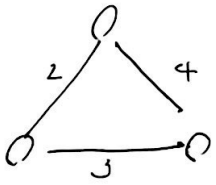
{ erasing all edges of label ∞
{ adding all edges of the complement graph Γ^c and label them 2.

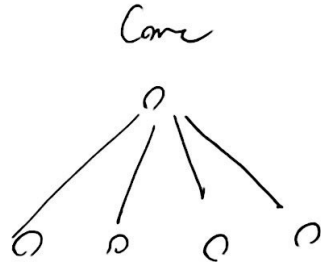
$$E(\Gamma^c) = \underline{E}(k_{\#V(\Gamma)}) - E(\Gamma)$$

Thm (k-Oguni)

$\#(V(\Gamma)) \geq 3$, $\Gamma^{<\infty}_{2}$ is not a cone.

then A_Γ is acylindrically hyperbolic.





Previous Works.

- Charney and Morris-Wright 2016
  for $\Gamma^{<\infty}_{2}$ is not a join.