μ-Zariski pairs of tork.

$f: \mathbb{C}^n, 0 \to \mathbb{C}, 0$

$V = f^{-1}(0)$ $\exists \varepsilon > 0.$ $0 < \delta \ll \varepsilon.$

$f: f^{-1}(D_\delta^*) \cap B_{\varepsilon_f} \to D_\delta^*$

$h_* : H_j(F)\,\mathbb{Q}$

$\zeta(t) = \prod_{i=0}^{n-1} P_i(t)^{(-1)^{i+1}}$

$P_i(t) = \det(h_t - t\,\mathrm{id}, H_j(F)\,\mathbb{Q})$

$\Rightarrow \zeta(t) = (1-t)^{-1} P_{n-1}(t)^{(-1)}.$

A'Campo m_i = multiplicity of $\pi^* f$.

$\pi: X \to \mathbb{C}^n, 0.$

be a good situation

$\pi.$

$\pi: X - \rho^{-1}(V) \to \mathbb{C}^n - V$

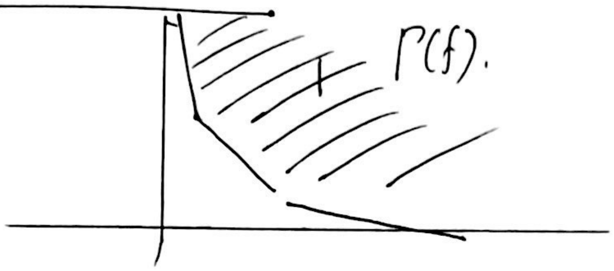
$\pi^{-1}(V) = \tilde{V} \cup E_1 \cup \dots \cup E_n$

$E_i^* = (E_i \setminus (\bigcup_{j \neq i} E_j \cup \tilde{V})) \cap \pi^{-1}(0)$

Th $\zeta(t) = \prod_i (1-t^{n_i})^{-\chi(E_i^{\circ})}$. 3 - ②.

(A'Campo)

Varchenko formula.



$$f = \sum a_\nu z^\nu$$

Newton bd

$$f_\Delta := \sum_{\nu \in \Delta} a_\nu z^\nu \qquad P = \begin{pmatrix} p_1 \\ \vdots \\ p_n \end{pmatrix} \in \mathbb{N}^n$$

weight vector.

$$\begin{tikzcd} l_p : \Gamma(f) \arrow[r] & \mathbb{R}_+ \\ \nu \arrow[r, mapsto] \arrow[u, phantom, "\in" sloped] & \sum p_i \nu_i \end{tikzcd}$$

$$\deg_p z^\nu = \sum p_i \nu_i$$

$$f_p = \sum.$$

---

f is Newton non-deg.

$\Longleftrightarrow \forall p \gg 0 \quad f_p : (\mathbb{C}^*)^n \longrightarrow \mathbb{C}$ no

Ex. $f = x^5 + x^2(x^2-y^2) + y^5$.

$P = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\quad f(x,y) = x^5 + x^2(x^2-y^2) + y^5$

f_P: ~~$x^5 + x^2(x^2 - y^2)$~~ $\mathbb{C}^{*2} \to \mathbb{C}$ 3 - ③.

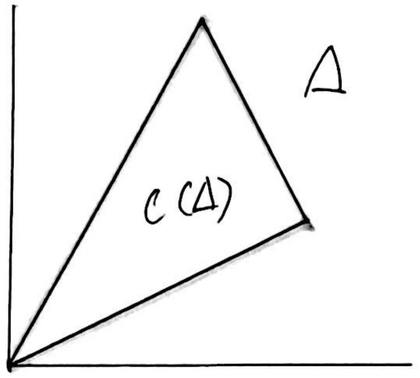
$x^5 + x^3(x^2 - y^2)$.   <u>Newton boundary</u>:

Th(Vanchanko).

$\forall I \subset \{1, \dots, n\}$.

$\mathfrak{F}_I = \{P :$ primitive weights for maximal faces of $f^I\}$

$f^I = f|_{\mathbb{C}^I} : \mathbb{C}^I \to \mathbb{C}$.



$C(\Delta) = \mathrm{Cone}(\Delta, 0)$.

$$\zeta(t) = \prod_I \zeta_I(t).$$

$$\zeta_I(t) = \prod_{P \in \mathfrak{F}_I} (1 - t^{d(P_i, f^I)})^{-\chi(P)}.$$

$$\chi(P) = (-1)^{I-1} (I)! \, \mathrm{vol}_{(I)} C(\Delta(P)) / d(P, f^I)$$

Eg. $f = x^2 + y^3 + z^5$ (E_8).

$I = \{\{1,2\}, \{2,3\}, \{1,3\}, \{1\}, \{2\}, \{3\}\}$
$\{1,2,3\}\}$.

$P = \begin{pmatrix} 15 \\ 10 \\ 6 \end{pmatrix}$, $d = 30$, $\zeta_I = (1-t^{30})^{-1}, (1-t^6)^{-1},$
$(1-t^{10})^{-1}, (1-t^{15})^{-1}$
$(1-t^2)^{-1}, (1-t^3)^{-1}$
$(1-t^5)^{-1}$

$$\zeta(t) = \frac{(1-t^{6})(1-t^{10})(1-t^{15})}{(1-t^{30})(1-t^{2})(1-t^{3})(1-t^{5})}.$$

$$\deg\zeta = 6+10+15-(30+2+3+5) = -9.$$

$\|$

$-1+(-1)^{n}\cdot\mu$

Def. $f(z)$ almost-Newton-non-deg.

$\exists\, S_0 \subset S_{\{1,\dots n\}}$. s.t.

f is non deg on $\forall\Delta$, $\dim\Delta < n-1$.

or

$\Delta \in S \setminus S_0$.

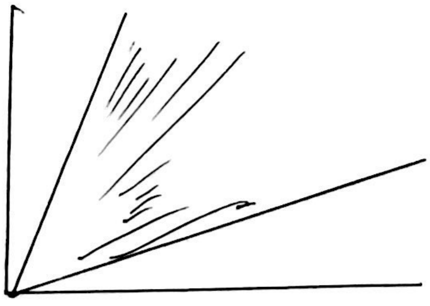
$\Gamma^*(f)$: <u>dual Newton diagram</u>.

$\mathcal{N}$:

$\Downarrow$

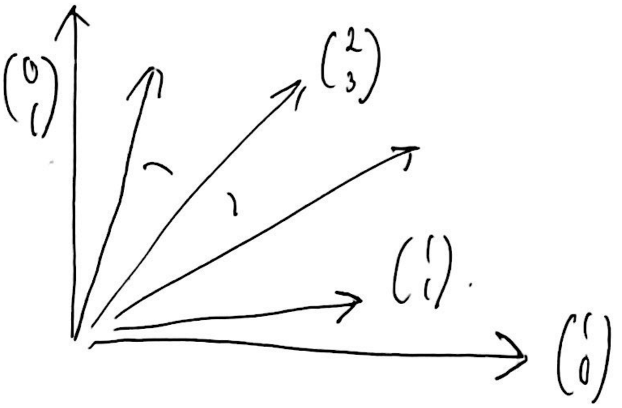
P, Q. $\quad P \sim Q \iff \Delta(P) = \Delta(Q)$.

canonical polyhedral decomposition



$\Gamma^*(f) \subset \Sigma^*$.

regular simplical



Σ^*: given

$\pi : X \to \mathbb{C}^n, 0$

toric modif.

Exceptional deivisor.

$\Leftrightarrow P \in \mathrm{Vertex}(\Sigma^*)$

of poihere

Th(-) f: almost non-deg.

$$\zeta(t) = \zeta^{(0)}(t) \underbrace{\zeta^{(Er)}(\tau)}_{\text{Error term.}} \prod_{P \in \Sigma V} \zeta_P(\tau).$$

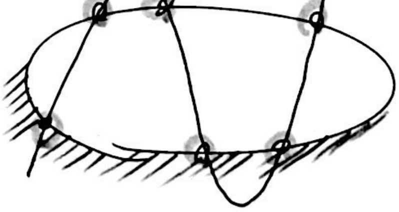
where $\zeta^{(Er)}(\tau) = \prod_{P \in S_0} (1-\tau^{d(P;f)})^{(-1)^n \mu(P)}$.

Def. Zariski pair of curves.

$\mathbb{P}^2 \supset C_1, C_2$.

$\Sigma(C_1), \Sigma(C_2) = 6A_2$.

$C_1 : f_2^3 + f_3^2 = 0$



$\exists Q$ conic.

$Q \supset \Sigma(C_2)$.

$\pi_1(\mathbb{P}^2 - C_1) \cong \mathbb{Z}_2 * \mathbb{Z}_3$

$\pi_1(\mathbb{P}^2 - C_2) \cong \mathbb{Z}_6$.

C_1, C_2 degree d.

Zariski pair of.

$$\bar{\Sigma} C_1 \xrightarrow{\varphi} \bar{\Sigma} C_2.$$

$$(C_1, P_1) \cong (C_2, P_2).$$

$^{\exists}\tilde{\varphi} : {}^{\exists}N(C_1) \to N(C_2)$ hence. $^{\nexists}\varphi : (\mathbb{P}^2 : C_1) \to (\mathbb{P}^2 : C_2)$.

Example. C_1, C_2 sextic. （コニック、ジェネーティック）

torus type non Tsuchihyn

$\exists$ 19 types of sextics of torus.

with simple sing.

Def. $f_1, f_2 : \mathbb{C}^n_0 \to \mathbb{C}_0$

isolated sing

μ - Zariski - pair of links

$$\mu(f_1) = \mu(f_2)$$

$$\zeta_{f_1}(t) = \zeta_{f_2}(t).$$

$n=3$.

Def f_1, f_2 Zariski pair of curves   degree of.

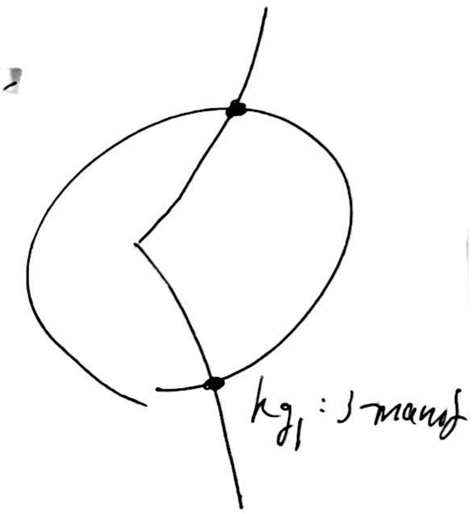
$$g_i = f_i(z) + z_3^{d_i n} \quad (i=1,2).$$

~~Thm~~

~~Ex~~ Conj $\{g_1, g_2\}$ define. μ Zariski pairs. of links

Th ① for selection of 19 types $\{g_1, g_2\}$ is a Zariski pair of links,

② $k_{g_1} \cong k_{g_2}$ diffeo



why? $\exists \varphi_0\ X \to V(g_1)$

same resolution graph.

$$\exists ?\ \varphi : S^5 \to S^5$$
$$\cup$$
$$k_{g_1} \cong k_{g_2}$$

B-function

A description of monodromic mixed Hodge modules. Takahiro. S-

MHM

mixed Hodge module

Hodge theory
Griffith's theory
Deligne's.

\+

D-modules
("singular obj")

Hodge theory.

↓

singular

=

MHM.

X: compact kahler mfd.

$w \in \mathbb{Z}$

$$H^{w}(X;\mathbb{C}) = H^{w}_{dR}(X)$$

$$= \bigoplus_{p+q=w} H^{p,q}$$

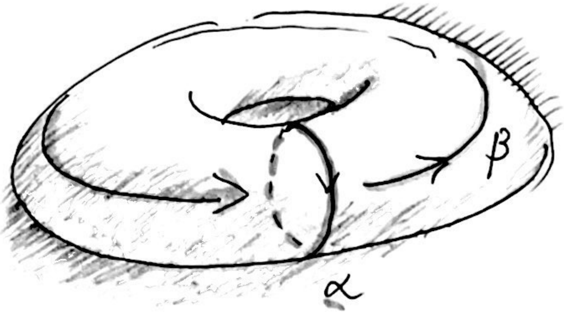
e.g X: elliptic curve.

$$H^{1}(X;\mathbb{C}) = \Omega^{1}(X) \oplus \overline{\Omega^{1}(X)}$$

~~→ period at~~ X.

$(H^1(X;\mathbb{C}), \Omega^1 \oplus \overline{\Omega^1}, H^1(X;\mathbb{Z}))$

→ period at X



→ X

recoverse

Fact (Deligne)

X : alg var/$\mathbb{C}$ $\qquad (W_\cdot = \{W_r\}_{r\in\mathbb{Z}})$

$H^n(X;\mathbb{C})$ has a canonical mixed Hodge str.

(i.e. $\exists W \subset H^n(X;\mathbb{C})$: weight filt

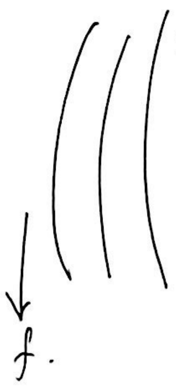
s.t. W_r/W_{r-1} : Hodge str of wt r)

Griffiths's theory

$f : Y \to D.$

$\cap$

$\mathbb{C}$



$f^{-1}(t) \sqcup$

$H^n(f^{-1}(t);\mathbb{C})$ Hodge str.

$\cup$

$F^\cdot H^n(f^{-1}(t))$.

ν : holomorphic bdle with flat connection $\qquad H^n(f^{-1}(t);\mathbb{C})$