$$Z_p(X.T) = \frac{\prod_{i=1}^{3} P_p^{2i-1}(T)}{\prod_{i=0}^{3} P_p^{2i}(T)}$$

$$L_i(X,s) = L(H^i_{et}(\overline{X}, \mathbb{Q}_\ell), s).$$

$$\rho^i_{X,\ell} : G_{\mathbb{Q}} \to G$$

ホッジタイプ (1.1.1.1).

$$L_i(X,s) := L(H^i_{et}(\overline{X}, \mathbb{Q}_\ell), s).$$

$$:= (*) \prod_{p \neq \ell : \text{good}} P^i_p(p^{-s})^{-1} \times (\text{factor corresponding to } \ell = p).$$

$$\xrightarrow{i=1} \zeta(s).$$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{11} & & 0 & \\ 1 & & 0 & & 0 & & 1 \\ & 0 & & h^{22} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

フロベニウス群のコホモロジー群への作用のトレース.

Are there global function that determine the L-function L(X,s)?
More correctly, are there certain (particular) forms that determine L(X.s)?

Def. A calabi-Yau -

(2)

$$\begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 1 & & h^{1,1} & & 1 \\ & 0 & & 0 & \\ 0 & & h^{2,2} & & 0 \\ & & 1 & & \end{array}$$

Thm. $\boxed{\text{Every rigid Calabi–Yau three fold } X \text{ over } \mathbb{Q} \text{ is modular, that is, there is a modular form } f \text{ of weight } 4 = 3+1, \text{ on some } \Gamma_0(N), \text{ s.t. } L(X,s) = L \ldots}$

$f(q) = [\eta(q)\eta(q^2)\eta(q^3)\eta(q^6)]^2.$

$f(q) = [\eta(q)\eta(q^2)\eta(q^3)\eta(q^6)]^2$

$\eta(q) = \cancel{(\ )}\ (q,q) q^{1/24}$

$= q^{1/24}(1-q)(1-q^2)(1-q^3)\ldots$

$\underline{1+2+3+6.}$
$12.$

$(1-q^2)^2 = (1-2q^2 \cdot$

$\cancel{(1-q)(1+q)} = (1-2q+q^2)(1-q^2).$

$(1-1$

$= (1-q)^2(1-q^2)^2\ldots$

$(1-q^2)^2(1-q^4)\ldots$

$(1-q^3)^2(1-q^6)^2\ldots$

$(1-q^6)^2(1-q^{12})^2\ldots$

$= \underline{(q;q)_\infty^2 (q^2;q^2)_\infty^2 (q^3;q^3)_\infty^2 (q^6;q^6)_\infty^2} = \cancel{1-2q-2q^2\ldots}$

$= 1 - 2q - 3q^2 + 4q^3 + 6q^4 + 6q^5 - 16q^6 \ldots$

$_4F_3$: Gaussian Hyper Geometric Function. ③

conifold point, attractor point (string theory) → basically singular point.

$$P_p(T) = (1-\chi(p)pT)(1-a_pT+p^3T^2) \in \mathbb{Q}[T]$$

$$(1-2q^2+q^2)(1-2q^2+q^4)(1-2q^3+q^6)\cdots$$

$\underline{h^{p,q}=1,\quad (p,q),\ p+q=3,}$

E.g.

$Y_0^2 =$

$$P_p(T) = 1+a_1T+a_2T^2+p^3a_1T^3+p^6T^4 \in \mathbb{Z}[T]$$

Y_1^2

$$P_p(T) = 1+a_1T+a_2T^2+p^3a_1T^3+p^6T^4 \in \mathbb{Z}[T]$$

Y_2^2

Y_3^2

$L(X,s) =$

$\cancel{\eta = d_k^2}\quad 2^2\cdot 3^2\cdot 5^{-4}.$

$\eta = d_k^2 \cdot \mathrm{Norm}(\eta) = 2^2\cdot 3^2\cdot 5^4 = 36\cdot 25^2.$

$P_7 = \cancel{1-3x}$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & & & h^{1,1} & & & \\ 1 & & 0 & & 0 & & 1 \\ & & & h^{2,2} & & & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

Every rigid Calabi-Yau three fold. (4)

X over $\mathbb{Q}$ is modular, that is

there is a modular form f of

weight $4 = 3+1$

$\Updownarrow$ E_8 - Lattice.

Eisens

$$E_k(z) = \sum_{m,n \neq 0} \frac{1}{(mz+n)^k}$$

Calabi Yau	3	fold.
	|	
Modular Weight	4.	Eisenstein Series $\to E_4(z) = 1 + 240q^2 + \cdots$
	|	
Lattice	8	dim ?

2

Calabi Yau	?	fold. $\leftarrow$ Probably 5 ?
Modular Weight.	12.	$E_{12}(z) = 1 - 196560q^m$
Lattice.	24	dim (Leech).

$$\begin{array}{r} 600 \\ 150 \\ 6 \\ \hline 756 \end{array}$$

$$\left\{\begin{array}{l} E_{\cancel{8}4} - E_{\cancel{4}8}\text{-Lattice} \quad 240 \\ E_{12} - \Lambda_{24} \text{ Leech Lattice} \quad 196560. \end{array}\right.$$

$E_6 \overset{?}{-} \Lambda_{12}\ (252 \times 3),\ 756.$

$E_8 - \Lambda_{16}. - 4320.$

Elliptic Artin Groups. $\longrightarrow$ Elliptic Hecke Alg $H(R,G)$. 2 − ①.

$A(R,G)$. j-w Yoshihisa Saito

$\searrow$ Elliptic. Artin Monoids. $A^{+}(R,G)$.

Finite. root system.

$\alpha \in R \subset F$, root v. $\dim_{\mathbb{R}} F = l$.

$$W(R) = \langle w_\alpha \mid \alpha \in R \rangle \subset GL(V).$$

$$= \langle w_\alpha \in \Gamma \mid (w_{\alpha_i} w_{\alpha_j})^{m_{ij}} = 1 \rangle. \quad \Gamma = \{\alpha_1, \alpha_2, \alpha_3\}.$$

$\mathrm{Ham}_{\mathbb{R}}(F, \mathbb{C}) \supset \cup H_\alpha := \{\ker(\alpha)\}$

$$\downarrow \quad \Delta = \left(\frac{\partial(P_1 \cdots P_l)}{\partial(x_1 \cdots x_l)} \right)^2.$$

0

$\vdots$

Elliptic Root System.

F : a finite root system.

$I : F \times F \to \mathbb{R}$. symmetric.

$\alpha \in F$, $I(\alpha.\alpha) > 0$.

$\alpha^\vee := \dfrac{2\alpha}{I(\alpha.\alpha)}$, $w_\alpha : F \to F$. $w_\alpha(u) := u - I(\alpha^\vee, u)\alpha$

$w_\alpha^2 = 1$, (Involution).

$\mathrm{sign}(I) = (l, 0, 0)$.
$\Rightarrow R$: finite. root.

$\mathrm{sign}(I) = (\mu_+, \mu_0, \mu_-)$.

$R \subset F$ is a generalized root system.

0) $\forall \alpha \in R$. $I(\alpha.\alpha) > 0$.

i) $Q(R) = \sum_{\alpha} \mathbb{Z} R \subset F$.

ii) $\alpha.\beta \in R$ $\begin{cases} Q(R) \otimes_{\mathbb{R}} \mathbb{R} \xrightarrow{\sim} F. \\ I(\alpha^\vee, \beta) \in \mathbb{Z} \end{cases}$

iii). $\forall \alpha \in R$, $w_\alpha R = R$.

iv) $R = R_1 \amalg R_2$. $R_1 \perp R_2 \Rightarrow R_1 = \phi$ or $R_2 = \phi$

∘ $\mathrm{sign}(I) = (l, 2, 0)$. — rank2-トーラス

$\mu_0 = \dim_{\mathbb{R}}(\mathrm{rad}(I))$, $\mathrm{rad}(I) := F^{\perp}$.

$\mathrm{rad}(I) := F^{\perp}$.

$:= \{x \in F \mid I(x, y) = 0, \forall y \in F\}$.

$W(R) := \langle w_\alpha \mid \alpha \in R \rangle$. ~~does~~ not. proposing discontinuously.

on F not on F^*.

rank 2.

$W_f \ltimes (Q(R) \otimes \mathrm{rad}(I)_{\mathbb{Z}})$.

$\cong \mathbb{Z}^{2l}$.

semi direct. Group

半直群.

Elliptic. diagram.

$G \subset \mathrm{rad}(I) \subset F$.

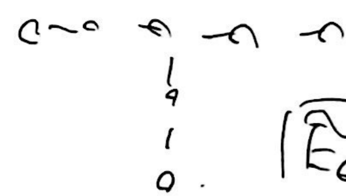
$\cong \quad \cong$

$\mathbb{R} \leftarrow \mathbb{R}^2$.

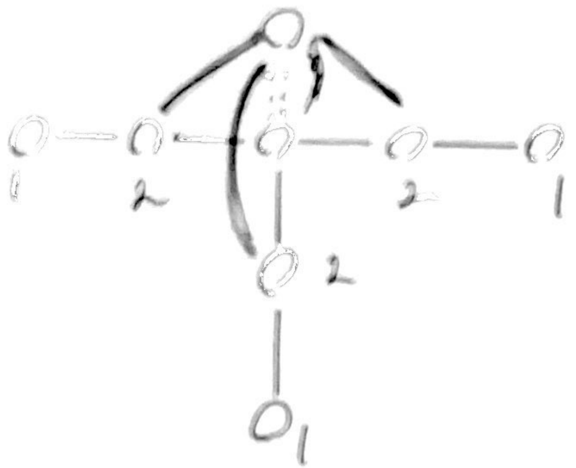
$\mathbb{R}/G \subset F/G$ affine

$\cup$ not system

Γ_{aff}

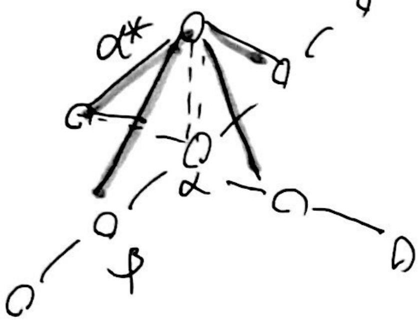


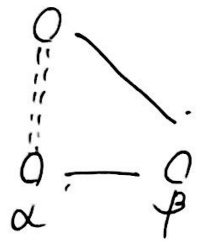
$\boxed{\tilde{E}_6}$.



$I(\alpha, \beta) = 2$

$\tilde{W}(RG) = \langle w_\alpha,\ \alpha \in \Gamma(RG) \mid$ elliptic Coxeter relation $\rangle$.





$\underset{w_\alpha}{\overset{\alpha}{\parallel}} \underset{w_\beta}{\overset{\beta}{\parallel}} w_\alpha = w_\beta w_\alpha w_\beta$

$\to b a a^* b a a^* = a a^* b a a^* b$.

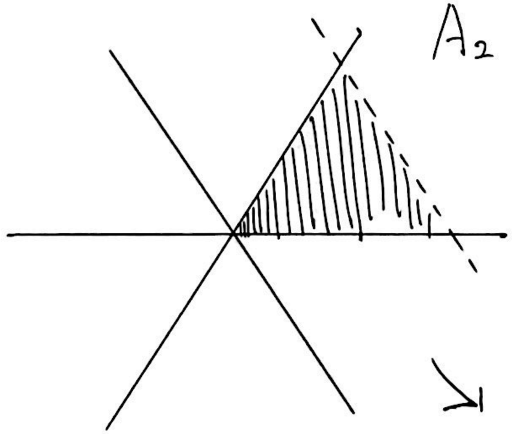
$\boxed{\pi_1(S \setminus D, *) = \langle a, b \mid aba = bab \rangle}$

$$\begin{array}{ccc} \bigcup_\alpha H_\alpha & \subset & \tilde{\mathbb{E}} \\ \downarrow & & \downarrow \\ D_{ell} & \subset & \tilde{\mathbb{E}} / \tilde{W}(R,G) \end{array}$$

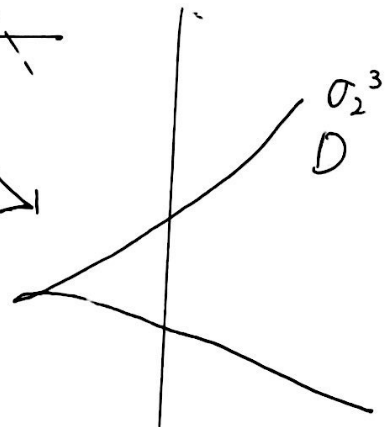
Thm

$$\boxed{\begin{array}{l} \pi_1(\tilde{\mathbb{E}}/\tilde{W}(R,G) \setminus D_{ell}, *) \\ \cong \langle a_\alpha,\ \alpha \in \Gamma(R,G) \mid \text{elliptic braid relation} \rangle \end{array}}$$

$\left(F^* \otimes \mathfrak{S}/W\right)$.

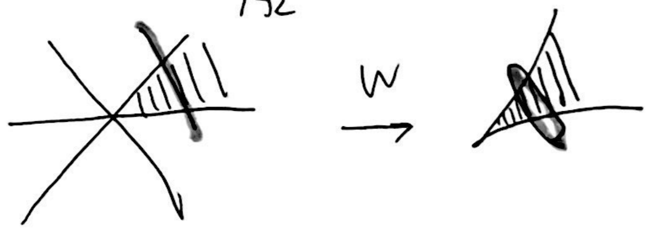


A_2



$\sigma_2^3 = \sigma_3^2$.

D



A_2

Q.1

Calabi Yau 3 fold

|

Modular Weight 4 → Eisenstein Series $E_4(z) = 1 + 240q^2 + 2160q^4 + \cdots$

| $= \Theta_{\Lambda_{E_8}}$.

E_8 Lattice. in 8 dimension

Then.

Calabi Yau ? fold.

|

Modular Weight 12 $\Theta_{\Lambda} = 1 + 196560q^2 + \cdots$

|

Λ_{24} Leech-Lattice. in 24 dim

Q2. Mock Theta と関係のある多様体は何か?

(Mock Modular eg $E_2(z)$) Mass form ? — manifold —

Q3. 高次元 Penrose Tailing にでの擬周期格子に対する Theta function は何か?